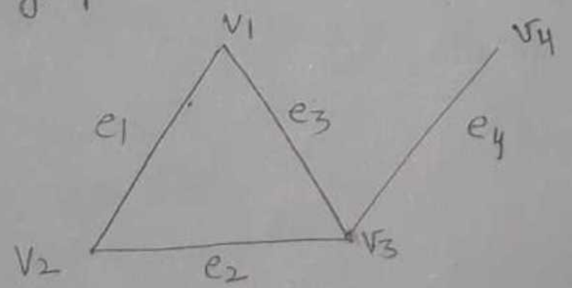
Graph Theory

1. (definition of Graph):- A graph G consist of two set :-
   (i) A non-empty set V whose elements are called vertices or node or points of G.
   The set V(G) is called the vertex set of G.
   (ii) A set E of edges such that each edge $e \in E$ associated with ordered or unordered pairs of of elements of V. The set E(G) is called the edge set of G.
   The graph G with vertices V and edges E is written as G(V, E).

2. Finite Graph :- A graph with finite ~~vert~~ vertex set is called a finite graph.

3. Simple graph :- A graph in which each edge connects two different vertices and where no two edges connect the same pair of vertices is called a simple graph. example.



The graph is a simple graph, whose vertex set is $V = \{v_1, v_2, v_3, v_4\}$ and set of edges is
$E = \{e_1, e_2, e_3, e_4\}$. Where $e_1 = (v_1, v_2)$

$e_2 = (v_2, v_3)$.

$e_3 = (v_1, v_3)$

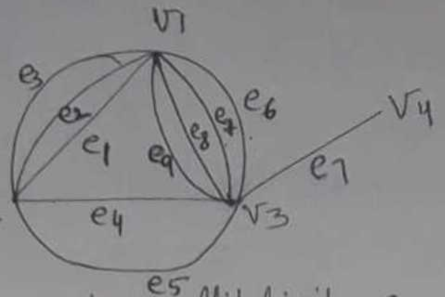
$e_4 = (v_3, v_4)$.

4. **Multi Graph** :- Graphs that have multiple edges connecting the same vertices are called multi graphs.

When there are different m edges associate to the same unordered pair ie of vertices $\{u, v\}$.

$\{u, v\}$ is an edge of multiplicity m.

example :-



$\{(v_1, v_2)\}$ is an edge of multiplicity 3.

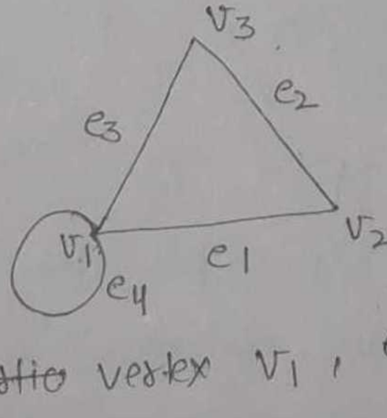
$\{(v_2, v_3)\}$ is an edge of multiplicity 2.

$\{(v_3, v_1)\}$ is an edge of multiplicity 4.

$\{(v_3, v_4)\}$ is an edge of multiplicity 1.

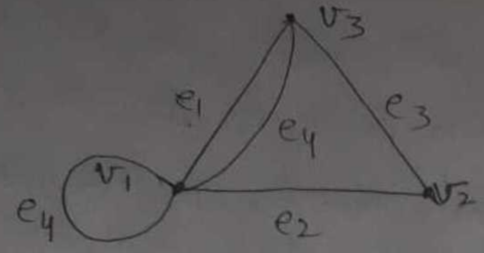
5. **Loop** :- A loop or (self loop) is an edge for which both end vertices coincide. example.

~~T~~ In graph G, there are three vertices and four edges.



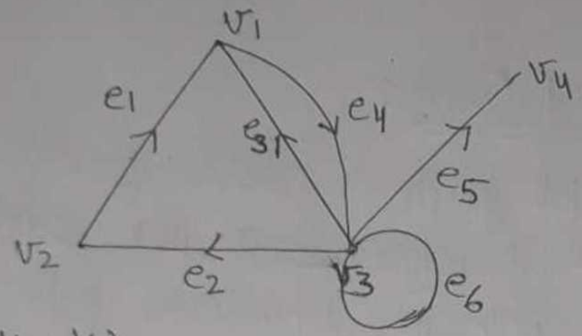
~~ka~~ here at ~~vestio~~ vertex v_1, e_4 represent loop.

. **Pseudo graphs** :- Graphs that may include loops and possibly multiple edges connecting the same pair of vertices are called Pesudo Graphs, example



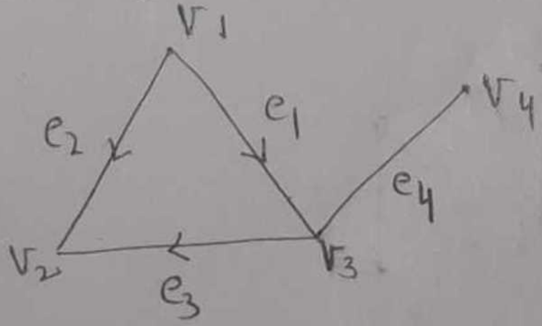
7. **Directed Graph :-** A directed graph (or digraph) (V, E) consists of a non-empty set of vertices V and a set of directed edges (or arcs) E.
each directed edge is associated with an ordered pair of vertices. the directed edge associated with the ordered pair (u, v) is said to start from u and end at v.

example.

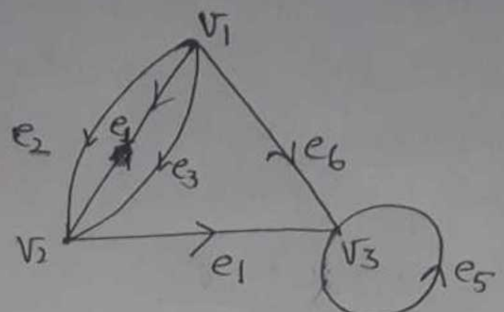


$e_1 = (v_2, v_1)$
$e_2 = (v_3, v_2)$
$e_3 = (v_3, v_1)$
$e_4 = (v_1, v_3)$
$e_5 = (v_3, v_4)$
$e_6 = (v_3, v_3)$

• **Simple directed Graphs :-** When a directed graph has no loops and no multiple directed edges, it is called a simple directed graph.



9. Directed Multi Graphs :- Directed multigraphs that may have multiple directed edges ~~for~~ from a vertex to a second vertex. We called such graphs directed multigraphs. When there are m directed edges, each associated to an ordered pair of vertices (u, v), we say that (u, v) is an edge of multiplicity m. eg.

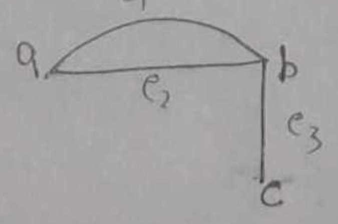


10. Undirected Graph :-

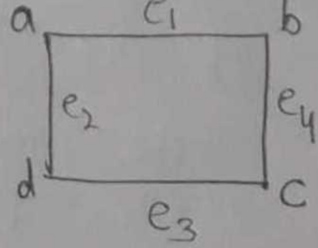
An undirected graph G consist of set V of vertices and a set E of edges such that each edge $e \in E$ is associated with an unordered pair of vertices we can refer to and edge joining the vertex pair i and j or j an i.

Question → State which of the following graphs are simple. If not, give the reason.

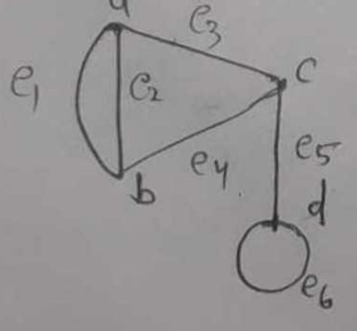
(i)



(iii)



(ii)



Soln:- (i) It is not a simple graph because it has multiple edges between vertices a and b.

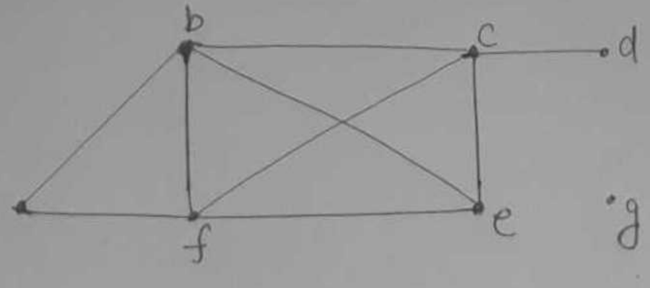
(ii) It is not a simple graph because multiple ~~be~~ between a and b.

(iii) It is a simple graph.

11. Degree of a vertex :- The degree of a vertex ~~u~~ in an undirected graph is the number of edges incident with it, except that a loop at a vertex contributes twice to the degree of that vertex u is denoted by deg u.

Q What are the degree of the vertices in a graph G.

(i)



(i) $deg(a) = 2$

$deg(b) = 4$

$deg(C) = 4$

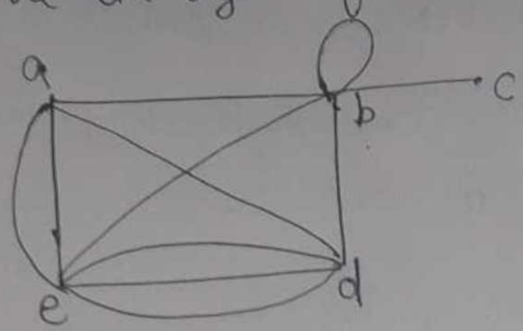
$deg(d) = 1$

$deg(e) = 3$

$deg(f) = 4$

$deg(g) = 0$

Q What are the degree of the vertices in a graph.



de (a) = 4

deg (b) = 6

deg (c) = 1

deg (d) = 5

deg (e) = 6